Hydrodynamic test problems

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This talk is concerned with verification

Verification is the process of confirming that the equations were coded correctly.

Validation is the process of confirming that the equations represent the correct physics.

Verification is the process of confirming that we are solving the equations correctly. Validation is the process of confirming that we are solving the correct equations.

We describe 3 classes of hydrodynamic test problems for an imploding capsule

The early phase: Concerned with 2 events:
(1) Initial shock propagation within the beryllium or glass capsule.
(2) The jump-off velocity, i.e. the initial velocity at the inner surface when the hydrodynamic shock first arrives there.

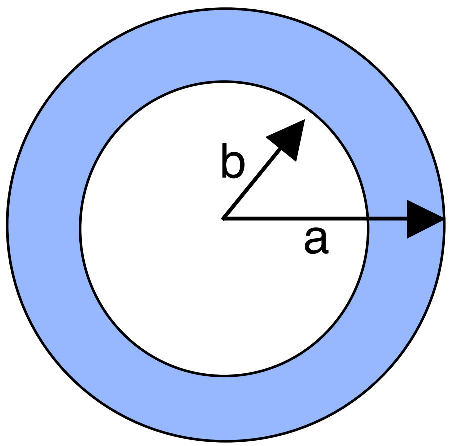
The late-time solution: Concerned with the velocity and displacement, as a function of time, at the inner surface of the capsule.

Perturbation growth: Concerned with the growth of perturbations on the inner and outer surfaces of the shell and the feed-through effect between both surfaces.

The test configuration is a spherical thin shell driven by an applied pressure on its outer surface

The shell has an outer radius a and an inner radius b and density ρ_0

The driving pressure is exponentially decaying with time.



This form was chosen for its simplicity and versatility.

$$P(t) = P_0 e^{-\alpha t} << K$$

Where K is the bulk modulus

Earlier work in geophysical applications is adaptable to the current problem

Relevant earlier work includes:

Sharpe, J. A. , "The production of Elastic Waves by Explosion Pressures, I Theory and Empirical Observations," Geophysics 7, 144-154 (1942).

Blake, F. G. Jr., "Spherical Wave propagation in Solid Media, "J. Accoust. Soc. Am. 24, pp211-214, (1952).

This earlier work can be adapted by considering a convergent (instead of a divergent) spherical wave and specializing to the case of a strength-less shell (σ = 1/2)

The displacement and velocity at any location R reduce to:

$$u(R,\tau) = \frac{P_0 a}{\rho_0 \alpha^2}\left[\frac{\alpha}{Rc}(e^{-\alpha\tau} - 1) - \frac{1}{R^2}(1 - e^{-\alpha\tau} - \alpha\tau)\right]$$

$$V(R,\tau) = -\frac{P_0 a}{\rho_0 \alpha}\left[\frac{\alpha}{Rc}e^{-\alpha\tau} + \frac{e^{-\alpha\tau}}{R^2} - \frac{1}{R^2}\right]$$

Where τ, the retarded time, is given by t - (a - R) / c and c is the longitudinal sound speed

The jump-off velocity is given by

$$V_j = 2P_0 a / bc\rho_0$$

The effects of small compressibility on the jump-off velocity can be accounted for

The main effect is to replace the sound speed c with the shock speed $Us = [(K + P_0) / \rho_0]^{1/2}$

We can further improve the estimate by taking into account the following two effects:

(1) The outer radius, a, decreases as the shock propagates into the shell.

(2) The peak pressure of a non-uniform shock wave decreases as a result of nonlinear propagation in the rarefaction following it. This effect is sometimes referred to as "hydrodynamic attenuation" and has been worked out by Duvall in 1962.

The late-time solution can be derived by equating the work done with the rate of change of kinetic energy

The rate of change of kinetic energy of the shell is :

$$\frac{d}{dt}\int_{R_i(t)}^{R_o(t)} \frac{1}{2} 4\pi\rho_0 R^2 [V(r)]^2 \, dR$$

The work done by the pressure is :

$$4\pi[R_o(t)]^2 P_0 e^{-\alpha t} V_o(t)$$

Conservation of volume implies that the particle velocity within the shell at location R is proportional to $1/R^2$

Differentiation and the chain rule yields an expression for the acceleration as a function of time.

The late-time solution can be reduced to two coupled first-order ordinary differential equations (ODE)

For an incompressible shell, the ODE's can be obtained for any applied pressure.

For the exponentially decaying pressure, the acceleration at the outer surface of the shell is given by the simple general expression:

$$a_o(t) = 0.5 \frac{\dfrac{-2P_0 e^{-\alpha t}}{\rho_0} + 3V_o^2(t) + rV_o(t)[rV_i(t) - 4V_o(t)]}{r(t)[R_o(t) - R_i(t)]}$$

Where the subscript *o* refers to values at the outer surface, subscript *i* refers to values at the inner surface, V(t) is the velocity, R(t) is the radius, and r(t) is the ratio $R_o(t)/R_i(t)$.

The late-time solution for the compressible shell can be obtained if the shell is accelerated shocklessly

We use the exponentially decaying pressure as the driving force and search for initial conditions that lead to shockless acceleration.

We would like the solution using these initial conditions to closely match the time averaged solution for the shock accelerated case.

We thus require that the total mass and thickness of the shell in the shockless case be identical to that in the shock accelerated case.

These initial conditions must have smoothly varying pressure and velocity across the shell at t = 0

These smoothly varying functions must also be consistent with the equation-of-state (EOS) of the shell and the exponentially decaying pressure on the outer boundary.

We start with the equations of momentum and mass conservation in Eulerian spherical coordinates:

$$\frac{\partial P}{\partial R} = -\rho \frac{DV}{Dt} \qquad \textbf{and} \qquad \frac{\partial V}{\partial R} + \frac{2V}{R} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

Where P is the pressure, V is the velocity, ρ is the density and

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + V\frac{\partial}{\partial R}$$

We restrict ourselves to the case of a simple yet useful EOS in which the pressure P is related to the compression through a bulk modulus K as follows: $P = K(\rho/\rho_0 - 1)$

The initial conditions leading to a shockless acceleration, are given by the solution of the following 3 coupled ODE

$$\frac{dP}{dR} = -\rho_0 a(R)[K + P(r)]/K$$

$$\frac{dV}{dR} = -\frac{2V}{R} + \frac{\alpha P(R)}{K + P(R)}$$

$$\frac{da}{dR} = -\frac{2a(R)}{R} - \frac{\alpha^2 KP(R)}{[K + P(R)]^2} + 2\frac{V^2}{R^2} + \left(\frac{dV}{dR}\right)^2$$

Subject to the boundary conditions that the pressure at the inner boundary be identically zero, the total momentum be identically zero, and the total mass be identical to the shock accelerated case.

Earlier work on perturbation growth includes:

Weir, S. T., E. A. Chandler, and B. T. Goodwin, “Rayleigh-Taylor Instability Experiments Examining Feedthrough Growth in an Incompressible, Convergent Geometry,” Physical Review Letters, Vol 80, p. 3763-6, April 1998.

Bakharakh, S. M., et al “Hydrodynamic Instability in Strong Media,” Report under LLNL-VNIIEF. LLNL report UCRL-CR-126710, March 1997.

Mikaelian, K. O., “Rayleigh-Taylor and Richtmyer-Meshkov instabilities and mixing in stratified shells,” Phys. Rev. A Vol. 32, No. 6, pp. 3400-3420, September 1990.

We have extended earlier work to include and exponentially decaying pressure

The perturbed radius R_{per} is given as a function of the unperturbed radius R_{unp} by the following equation:

$$R_{per} = R_{unp} + \eta(t,n,m) Y_{n,m}(\theta,\phi)$$

Where t is the time, $\eta(t,n,m)$ is the amplitude of the perturbation, and $Y_{n,m}(\theta,\phi)$ is the spherical harmonic.

the unperturbed radius is given by the late-time solution of the incompressible shell described earlier.

The analytic solution describe the evolution, in the linear regime, of each node, $\eta(t,n,m)$, in terms of two coupled ordinary second-order differential equations (Eq. 20 in Mikaelian 1990).

Which surface, if any, is more critical in manufacturing the capsule to reduce perturbations ?

We applied the perturbation to only one surface at a time. We found that for thin shells (such as the ones proposed for NIF), both surfaces are critical.

Perfecting only one surface will not stop the perturbations from feeding through to the other surface.

To account for the effect of compressibility, finely zoned 1D calculations can provide the average position, velocity, and acceleration at the inner and outer radii for a compressible shell. These can in turn be used in a modified Bell-Plesset equation to predict the growth of perturbations and compare with 3D code calculations with various resolutions.

Conclusions

The solution of the problems presented in this paper provide an independent check on the ability of newly developed 3D codes to model the hydrodynamics of a simplified implosion system.

Another benefit of these test problems is providing a way to compare the efficiency of various algorithms such as adaptive mesh refinement (Rendleman et al., 2000), or mesh free methods (Dilts, 2001).